POLYNOMIALS

BASIC CONCEPTS AND FORMULAE:

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polynomial	general form	number of zeroes
linear polynomial	$ax+b$	1
quadratic polynomial	ax^2+bx+c	2
cubic polynomial	ax^3+bx^2+cx+d	3
biquadratic polynomial	$ax^4+bx^3+cx^2+dx+e$	4

- Consider a quadratic polynomial ax^2+bx+c

  Degree = 2

  Maximum number of zeroes = 2

  Zeroes = $\alpha,\ \beta$

  Sum of zeroes, s = $\alpha+\beta$ = $\dfrac{-b}{a}$

  product of zeroes, p = $\alpha\beta$ = $\dfrac{c}{a}$

- A quadratic polynomial can have either two distinct zeroes or two equal zeroes (i.e. one zero) or no zeroes.

- Graph of polynomial ax^2+bx+c , $(a\neq 0)$ opens upwards like $\cup$ if $a>0$ and downwards like $\cap$ if $a<0$.

- Consider a cubic polynomial ax^3+bx^2+cx+d

  Degree = 3

  Maximum number of zeroes = 3

  Zeroes = $\alpha,\ \beta,\gamma$

  $\alpha+\beta+\gamma$ = $\dfrac{-b}{a}$

  $\alpha\beta+\beta\gamma+\alpha\gamma$ = $\dfrac{c}{a}$

  $\alpha\beta\gamma$ = $\dfrac{-d}{a}$

- Dividend = divisor × quotient + remainder

  $p(x)=g(x)\times q(x)+r(x)$

- Degree of $r(x)$ < degree of $g(x)$

SELECT THE CORRECT OPTION: (ONLY ONE OPTION IS CORRECT)

1) What is the LCM of x, x^2, x^3 and x^4?

(a) x (b) x^2 (c) x^3 (d) x^4

2) What is the HCF of x, x^2, x^3 and x^4?

(a) x (b) x^2 (c) x^3 (d) x^4

3) HCF of x^3y^2 and x^2y^3 is

(a) x^3y^2 (b) x^3y^3 (c) x^2y^2 (d) x^2y^3

4) Zeroes of $x^2 – 5x + 6$ are

(a) 2, 3 (b) –2, –3 (c) 6, –1 (d) –6, 1

5) Zeroes of $x^2 – 5x – 6$ are

(a) 2, 3 (b) –2, –3 (c) 6, –1 (d) –6, 1

6) Zeroes of $x^2 – x – 6$ are

(a) –2, 3 (b) 2, –3 (c) 6, –1 (d) –6, 1

7) Sum of zeroes of polynomial $x^2 – 2x – 3$ is

(a) 2 (b) 2/3 (c) 3 (d) –2

8) Sum of zeroes of polynomial $x^2 – 2 – 3x$ is

(a) 2 (b) 2/3 (c) 3 (d) –3

9) Polynomial with zeroes 3 and –5 is

(a) $x^2 + 2x + 15$ (b) $x^2 – 2x + 15$ (c) $x^2 – 2x – 15$ (d) $x^2 + 2x – 15$

10) The zeroes of $1061x^2 + 2121x + 2$ are

(a) both negative (b) both positive (c) one negative and other positive

(d) can’t be determined without calculation

11) The zeroes of $1061x^2 – 2121x + 2$ are

(a) both negative (b) both positive (c) one negative and other positive

(d) can’t be determined without calculation

12) The zeroes of $1061x^2 – 2121x – 2$ are

(a) both negative (b) both positive (c) one negative and other positive

(d) can’t be determined without calculation

13) How many zeroes are there for a bi-quadratic polynomial?

(a) 1 (b) 2 (c) 3 (d) 4

14) If α and β are the zeroes of $x^2 + x + 1$ then value of $\alpha^2 + \beta^2$ is

(a) 1 (b) –1 (c) 0 (d) 2

15) If α and β are the zeroes of $2x^2 - 2x - 1$ then value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is

(a) –2 (b) 2 (c) –0.5 (d) 0.5

16) If α and β are the zeroes of $x^2 - 2x - 2$ then value of $\frac{1}{\alpha^3} + \frac{1}{\beta^3}$ is

(a) –5 (b) 5 (c) 2.5 (d) –2.5

17) If α and β are the zeroes of $x^2 - px - (p + c)$ then value of $(1+\alpha)(1+\beta)$ is

(a) c – 1 (b) 1 – c (c) c (d) c + 1

18) If one zero of polynomial $ax^2 + bx + c$ is reciprocal of other, then

(a) ac = 1 (b) bc = 1 (c) a = c (d) b + a = 0

19) If zeroes of polynomial $ax^2 + bx + c$ are equal in magnitude but of opposite sign, then

(a) b = 0 (b) c = b (c) a = c (d) b + a = 0

20) What is the sum of the zeroes of polynomial $4(x^2 + 2x) - 3(x + 1) + x(1 + 2x)$?

(a) 1 (b) –1 (c) 3/4 (d) –3/4

21) By how many times (maximum) will $4x^2 + 7x^3 + x^6 + 1$ cut x–axis?

(a) 2 (b) 3 (c) 6 (d) none of these

22) α, β, γ are the zeroes of polynomial $x(x^2 - x + 1)$. What is the value of $\alpha\beta + \beta\gamma + \gamma\alpha$?

(a) 1 (b) –1 (c) 2 (d) 0

23) Find the polynomial if its zeroes are –3 and 1.

(a) $x^2 - 2x + 3$ (b) $x^2 + 2x - 3$ (c) $x^2 - 3x + 1$ (d) $x^2 + 3x + 1$

24) Find the polynomial if sum and product of its zeroes are –3 and 1.

(a) $x^2 - 2x + 3$ (b) $x^2 + 2x - 3$ (c) $x^2 - 3x + 1$ (d) $x^2 + 3x + 1$

25) Zeroes of polynomial $3x^3 - 7x^2 + 4x - 1$ are a – b, a, a + b. Find the value of 'a'.

(a) 7 (b) –7 (c) 7/9 (d) –7/9

26) Form a polynomial zeroes of which are $\frac{-3}{4}$ and $\frac{-4}{3}$ respectively.

(a) $x^2 - 3x + 4$ (b) $x^2 - 7x + 1$ (c) $12x^2 + 25x + 1$ (d) $12x^2 + 25x + 12$

27) What will be the remainder obtained on dividing $(x^4 + 3x + 4x^3 + 1)$ by $(x + 1)$?

(a) –5 (b) 5 (c) –3 (d) 7

28) If sum of zeroes of polynomial $x^3 + 2ax^2 + cx + b$ is –6, then 'a' equals

(a) 2 (b) 3 (c) –2 (d) –3

29) If sum of zeroes of polynomial $x^3 + 2ax^2 + cx + b$ is –6, then 'b' equals

(a) 2 (b) 3 (c) –12 (d) can't be determined

30) If one of the zeroes of cubic polynomial $ax^3 + bx^2 + cx + d$ is –1, then the product of other two zeroes is

(a) (b – a + c)/a (b) (b – a – c)/a (c) (a – b + c)/a (d) none of these

31) If α, β, γ are the zeroes of $ax^3 + bx^2 + cx + d$ then value of $\frac{1}{\alpha} + \frac{1}{\beta} + \frac{1}{\gamma}$ is

(a) –b/d (b) c/d (c) –c/d (d) –c/a

32) If α, β, γ are the zeroes of $ax^3 + bx^2 + cx + d$ then value of $\frac{1}{\alpha\beta} + \frac{1}{\beta\gamma} + \frac{1}{\gamma\alpha}$ is

(a) –b/d (b) b/d (c) –c/d (d) –c/d

33) If α, β, γ are the zeroes of $2x^3 + 3x^2 + x + 1$ then value of $\frac{\alpha}{\beta\gamma} + \frac{\beta}{\gamma\alpha} + \frac{\gamma}{\alpha\beta}$ is

(a) 7/2 (b) –5/2 (c) 1/6 (d) –5/4

34) If α, β, γ are the zeroes of $x^3 + 2x^2 + 3x + 4$ then value of $\alpha^2 + \beta^2 + \gamma^2$ is

(a) 2 (b) –2 (c) 6 (d) 10

35) **Polynomial ($ax^3 + bx + c$) is exactly divisible by ($x^2 + bx + c$), then 'ab' equals

(a) 1 (b) –1 (c) –1/c (d) 1/c

36) If the zeroes of $x^3 – 3px^2 + qx – r$ are in A.P., then

(a) $2p^3 = pq – r$ (b) $2p^3 = pq + r$ (c) $p^3 = \quad pq – r$ (d) $p^3 = 2pq + r$

37) A curve meets x-axis at 2, 1 and –2. Polynomial must be

(a) $x^3 – x^2 – 4x + 4$ (b) $x^3 – x^2 – 4x – 4$ (c) $x^3 – x^2 + 4x + 4$ (d) $x^3 + x^2 + 4x + 4$

38) A curve meets x-axis at 2, 1, –1, –2. Polynomial must be

(a) $x^4 – 5x^2 – 4$ (b) $x^4 – 5x^2 + 4$ (c) $x^5 – 5x^3 + 4x$ (d) $x^4 + 5x^2 + 4$

39) A curve meets x-axis at 2, 1, 0, –1, –2. Polynomial must be

(a) $x^4 – 5x^2 – 4$ (b) $x^4 + 5x^2 + 4$ (c) $x^5 – 5x^3 + 4x$ (d) $x^4 – 5x^2 + 4$

40) A line given by y = p(x) is parallel to x-axis. How many zeroes does it have?

(a) 0 (b) 1 (c) 2 (d) infinite

41) Number of zeroes in figure 1 is

(a) 0 (b) 1 (c) 2 (d) 3

42) Number of zeroes in figure 2 is

(a) 0 (b) 1 (c) 2 (d) 3

43) Polynomial of figure 3 is

(a) y = 0 (b) x = 0 (c) y = 1 (d) x = 1

44) Polynomial p(x) of figure 4 is

(a) $x^3 – x + 2$ (b) $x^3 + \quad 2x^2 – x – 2$ (c) $x^3 – 2x^2 – x + 2$ (d) $x^3 + 2x^2 – x + 2$

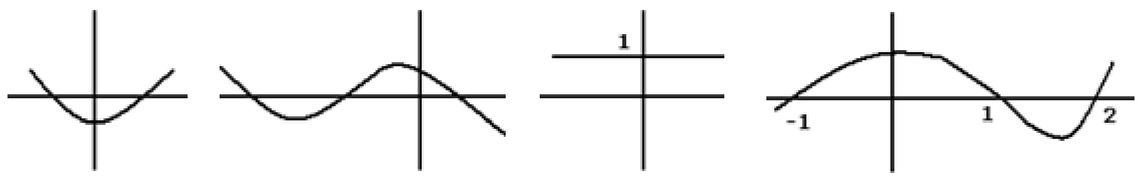


figure 1 figure 2 figure 3 figure 4

45) Let L and H be the LCM and HCF of two numbers A and B. Now which of the following is true?

(a) L + H = A + B (b) L – H = A – B (c) L/H = A/B (d) LH = AB

46) A real number ‘a’ is a zero of the polynomial f(x) if

(a) f(a) = 0 (b) f(–a) = 0 (c) f(a) > 0 (d) f(–a) < 0

47) The zeroes of polynomial f(x) are the coordinates of the points where the graph intersects

(a) x-axis (b) y-axis (c) origin (d) (x, y)

48) If 3 is the zero of f(x) then which of the following is the factor of f(x)?

(a) x – 3 (b) x + 3 (c) 3x – 1 (d) 3x

49) How many maximum zeroes can a quadratic polynomial have?

(a) 0 (b) 1 (c) 2 (d) 4

50) How many maximum zeroes can a bi-quadratic polynomial have?

(a) 0 (b) 1 (c) 2 (d) 4

51) Polynomial with sum and product of zeroes s and p respectively is given by

(a) $x^2 - sx + p$ (b) $x^2 + sx - p$ (c) $x^2 + sx + p$ (d) $x^2 - sx + p = 0$

52) If zeroes of polynomial $ax^2 + bx + c$ are reciprocal of each other then

(a) a = c (b) ac = 1 (c) ac + 1 = 0 (d) a + c = 0

53) If zeroes of polynomial $ax^2 + bx + c$ are equal in magnitude but are of opposite sign then

(a) b + c = 0 (b) b = 0 (c) a = 0 (d) a = b = c

54) The graph of $y = 9x^2 - 6x + 1$ cuts x – axis at

(a) $0, \frac{1}{3}$ (b) $\frac{1}{3}$ and $\frac{1}{3}$ (c) $\frac{-1}{3}$ and $\frac{1}{3}$ (d) do not cut

55) The zeroes of $4\sqrt{3}x^2 + 5x - 2\sqrt{3}$ are

(a) $\frac{2}{\sqrt{3}}, \frac{-\sqrt{3}}{4}$ (b) $\frac{-2}{\sqrt{3}}, \frac{\sqrt{3}}{4}$ (c) $\frac{-2}{\sqrt{3}}, \frac{-\sqrt{3}}{4}$ (d) none of these

56) The sum of squares of zeroes of $f(x) = x^2 - 8x + k$ is 40. What is the value of ‘k’?

(a) –12 (b) 12 (c) 24 (d) none of these

57) If α and β are the zeroes of $f(x) = x^2 + x + 1$, value of $\frac{1}{\alpha} + \frac{1}{\beta}$ is

(a) 0 (b) 1 (c) – 1 (d) none of these

58) The cubic equation with zeroes 1, 2, 3 is

(a) $x^3 + 6x^2 + 11x + 6$ (b) $x^3 + 6x^2 + 11x - 6$ (c) $x^3 - 6x^2 + 11x - 6$ (d) $x^3 - 6x^2 - 11x - 6$

59) **What will be the sum of zeroes of the polynomial $x^2 + 3x^5 + 12x^3 - x + 2$?

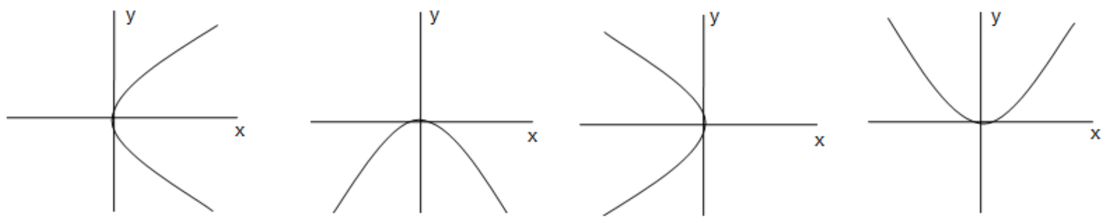
(a) – 3 (b) –0.25 (c) 17 (d) none of these

60) If two zeroes of $f(x) = 2x^4 + x^3 - 14x^2 - 19x - 6$ are – 1 and – 2, then other two zeroes are

(a) 0.5, –3 (b) –0.5, 3 (c) –0.5, –3 (d) none of these

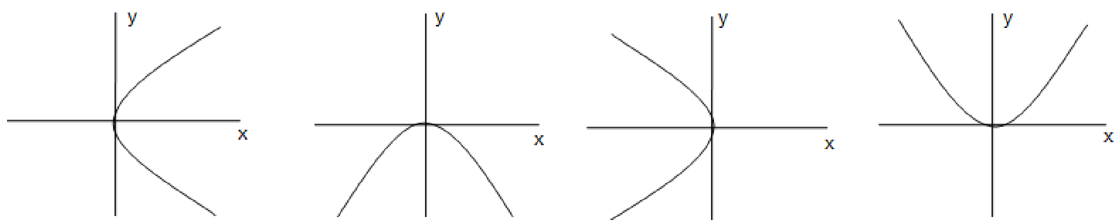
61) **What can be the curve of $y = x^2$?

(a) (b) (c) (d)



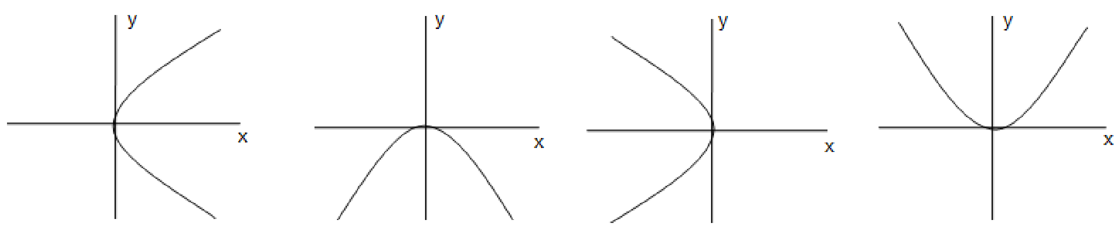
62) **What can be the curve of $y = -x^2$?

(a) (b) (c) (d)



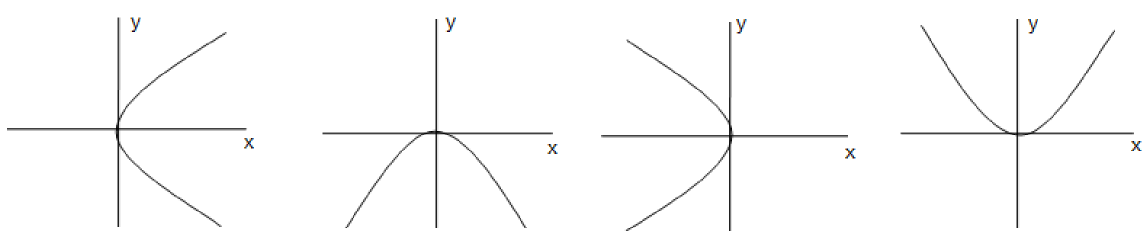
63) **What can be the curve of $x = y^2$?

(a) (b) (c) (d)



64) **What can be the curve of $x = -y^2$?

(a) (b) (c) (d)



SELECT THE CORRECT OPTION(S) (MORE THAN ONE OPTION CAN BE CORRECT)

(not meant for examination point of view)

65) If α and β are the zeroes of f(x) = $x^2 + 3 - 4x$, then

(a) $\alpha+\beta=-3$ (b) $\alpha\beta=-4$ (c) $\alpha+\beta=4$ (d) $\alpha\beta=3$

66) If α and β are the zeroes of f(x) = $x^2 - 3x + 2$, then

(a) $\alpha^2+\beta^2=5$ (b) $\alpha^3+\beta^3=9$ (c) $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=5$ (d) $\frac{1}{\beta}+\frac{1}{\alpha}=\frac{1}{5}$

67) Which of the following is a zero of polynomial ($2x^4 - 3x^3 - 3x^2 + 6x - 2$)?

(a) $\sqrt{2}$ (b) $-\sqrt{2}$ (c) 1 (d)1/2

68) Consider a polynomial p(x) = $8x^4 + 14x^3 - 2x^2 + 7x - 8$ and Q(x) = $4x^2 + 3x - 2$. Which of the following is true?

(a) If 14x + 10 is added to p(x) then the polynomial obtained is divisible by Q(x).

(b) If 14x – 10 is added to p(x) then the polynomial obtained is divisible by Q(x).

(c) If 14x + 10 is subtracted from p(x) then the polynomial obtained is divisible by Q(x).

(d)If 14x – 10 is subtracted from p(x) then the polynomial obtained is divisible by Q(x).

69) If $x^4 + x^3 + 8x^2 + ax + b$ is exactly divisible by $x^2 + 1$, then

(a) a + b = 8 (b) 2a – b + 6 = 0 (c)3a – b + 4 = 0 (d)a = 7b

70) If α , β , γ are the zeroes of $2x^3 - 3x^2 + 4x + 1$ then

(a) $\alpha+\beta+\gamma=\frac{3}{2}$ (b) $\alpha\beta+\beta\gamma+\gamma\alpha=2$ (c) $\frac{1}{\alpha}+\frac{1}{\beta}+\frac{1}{\gamma}=\frac{2}{3}$ (d) $\alpha\beta\gamma=\frac{1}{2}$

SUBJECTIVE PART

71) Find the zeroes of the polynomial $p(x)=2x^2+5x+3$ and verify the relation between zeroes and the coefficients of the polynomial.

72) Two zeroes of a polynomial $2x^4-5x^3-11x^2+20x+12$ are 2 and –2. Find other two zeroes.

73) Two zeroes of a polynomial $2x^4-3x^3-3x^2+6x-2$ are $\sqrt{2}$ and $-\sqrt{2}$. Find other two zeroes.

74) Two zeroes of a polynomial $3x^4+6x^3-2x^2-10x-5$ are $\pm\sqrt{\frac{5}{3}}$. Find other two zeroes.

75) On dividing $3x^3 - 2x^2 + 5x - 5$ by polynomial $p(x)$, the quotient and remainder obtained are $x^2 - x + 2$ and -7 respectively. Find p(x).

76) On dividing polynomial $p(x)$ by $x^2 + 2x - 1$, the quotient and remainder obtained are $x^2 + x + 1$ and $2x + 2$ respectively. Find $p(x)$.

77) Find the value of k for which the polynomial $x^4 + 10x^3 + 25x^2 + 15x + k$ is exactly divisible by $x + 7$.

78) For what value of x will $x^4 - 3x^3 + 7x^2 - 8x + 9$ be exactly divisible by $x^2 - x + 4$.

79) What must be subtracted from $4x^4 + 2x^3 - 8x^2 + 3x - 7$ so that it may be exactly divisible by $2x^2 + x - 2$?

80) What must be added to $6x^5 + 5x^4 + 11x^3 - 3x^2 + x + 5$ so that it may be exactly divisible by $3x^2 - 2x + 4$?

81) If the polynomial $x^4 - 6x^3 + 16x^2 - 25x + 10$ is divided by another polynomial $x^2 - 2x + k$, the remainder obtained is $x + a$. Find the values of k and a.

82) If α and β are the zeroes of quadratic polynomial $x^2 - (k+6)x + 2(2k-1)$. Find the value of k if $\alpha + \beta = \frac{1}{2}\alpha\beta$.

83) One zero of the polynomial $3x^2 - 8x + 2k + 1$ is seven times the other. Find the value of k.

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(not meant for examination point of view)

84) If α, β, γ are the zeroes of $x^3 + 2x^2 + 3x + 4$ then find the value of $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2}$.

85) If the zeroes of the polynomial $x^3 - 3x^2 + x + 1$ are $a - b$, a, $a + b$, find the values of 'a' and 'b'.

86) Two zeroes of polynomial $x^4 - 6x^3 - 26x^2 + 138x - 35$ are $2 \pm \sqrt{3}$. Find other two zeroes.

87) Two zeroes of polynomial $x^4 - 2x^3 - 14x^2 + 10x + 21$ are $3 \pm \sqrt{2}$. Find other two zeroes.

88) If polynomial $6x^4 + 8x^3 + 17x^2 + 21x + 7$ is divided by another polynomial $3x^2 + 4x + 1$, the remainder comes out to be $ax + b$. Find the values of a and b.

89) Find the value of a and b if $p(x) = 6x^4 + 8x^3 - 5x^2 + ax + b$ is exactly divisible by $2x^2 - 5$.

answers

1)d	2)a	3)c	4)a	5)c	6)a	7)a	8)c	9)d	10)a
11)b	12)c	13)d	14)b	15)a	16)d	17)b	18)c	19)a	20)b
21)c	22)a	23)b	24)d	25)c	26)d	27)a	28)b	29)d	30)c
31)c	32)b	33)b	34)b	35)b	36)a	37)a	38)b	39)c	40)a
41)c	42)d	43)c	44)c	45)d	46)a	47)a	48)a	49)c	50)d
51)a	52)a	53)b	54)b	55)b	56)b	57)c	58)c	59)d	60)b
61)d	62)b	63)a	64)c	65)cd	66)ab	67)abcd	68)d	69)ac	70)ab

71) $-1, -\frac{3}{2}$ 72) $3, -\frac{1}{2}$ 73) $1, \frac{1}{2}$ 74) $-1, -1$

75) 3x + 1 76) $x^4 + 3x^3 + 2x^2 + 3x + 1$ 77) –91

78) –5 79) 5x – 11 80) 17x – 17 81) k = 5, a = –5

82) k = 7 83) 2/3 84) –7/16 85) $a = 1,\ b = \pm\sqrt{2}$

86) 7, –5 87) –1, –3 88) a = 1, b = 2 89) a = –20, b = –25